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COMPARISON OF THE BEST KNOWN FRACTURE CRITERIA WITH DATA ON NAT--ETC(U)  
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COMPARISON OF THE BEST KNOWN FRACTURE CRITERIA  
WITH DATA ON NATURALLY OCCURRING CRACKS.

9 *Author's reply*  
by  
10 S.B. Batdorf

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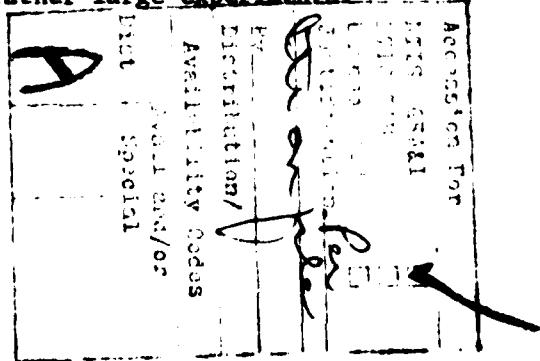
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INTRODUCTION

The concept of fracture as a result of crack instability was introduced by Griffith, who suggested two principles for determining the onset of catastrophic crack growth [1,2]. One was that fracture occurs when the strain energy released by crack extension is equal to the energy absorbed by the resulting free surfaces. The other was that fracture occurs when the tensile stress at the most highly loaded point on a crack surface reaches a critical value. The majority of criteria in use today for mixed mode fracture are related to one or the other of these two principles. One exception is a crude but convenient approximation which assumes that only the component of stress normal to the crack plane contributes to fracture. Others include the relatively recent energy density theory of Sih [3] and the equivalent specific energy theory of Gillemot [4].

A widely used experimental test of fracture criteria is to measure the strength and direction of extension of a known crack subjected to simple tension as a function of its orientation. Unfortunately, the predictions of several criteria are within the rather large experimental scatter, making a choice difficult.



Another type of experiment that can be used to compare fracture criteria employs unknown, naturally occurring cracks, and is statistical in nature. A major objective of statistical theories of fracture is to use the experimentally established fracture behavior in simple tension to predict fracture behavior in some other stress state. Since the result depends on the fracture criterion assumed [5], comparison of the prediction with experiment can be used to evaluate the relative merits of fracture criteria. A convenient second stress state for this purpose is equibiaxial tension. The purpose of the present paper is to carry out such a comparison.

#### THEORY

Assume that failure can occur due to any one of many independent and mutually exclusive mechanisms or causes, each having the infinitesimal probability of failure  $(\Delta P_f)_i$ . Under these circumstances, the probability of survival can be shown to be [5]

$$P_s = \exp[-\sum(\Delta P_f)_i] \quad (1)$$

In the case of a body containing Griffith cracks (open or closed elliptical cylinders) randomly oriented about axes normal to the stress plane and subjected to a uniform state of stress  $\Sigma$

$$(\Delta P_f)_i = \left( \frac{\omega}{\pi} \right) \left( V \frac{dN(\sigma_c)}{d\sigma_c} \Delta\sigma_c \right) \quad (2)$$

Here  $V$  = volume

$\sigma_c$  = critical stress, defined as the remote tensile stress that will cause fracture when applied normal to the crack plane.

$N(\sigma_c)$  = number of cracks per unit volume with critical stress  $\sigma_c$ .

$\omega(\Sigma, \sigma_c)$  = angle within which normal to crack plane must lie to cause fracture.

The right hand side of (2) is simply the probability that a crack in the critical stress range  $\Delta\sigma_c$  is present times the fraction of such cracks with an orientation that will result in fracture. Inserting (2) into (1) and going to the limit of small  $\Delta\sigma_c$

$$P_s(\Sigma) = \exp \left[ -V \int_0^\infty \frac{\omega}{\pi} \frac{dN}{d\sigma_c} d\sigma_c \right] \quad (3)$$

We note that the integral in (3) is finite because  $\omega = 0$  when  $\sigma_c > \sigma_1$ , the maximum principal stress (in some cases, to be discussed later,  $\sigma_c$  may exceed  $\sigma_1$  by several percent for  $\omega$  to be zero).

In the case of equibiaxial tension

$$\omega = \pi \quad (\sigma_c < \sigma) \quad (4a)$$

$$= 0 \quad (\sigma_c > \sigma) \quad (4b)$$

As a result, (3) reduces to the simple form

$$P_s(\sigma, \sigma) = \exp[-VN(\sigma)] \quad (5)$$

To facilitate the comparison between uniaxial and equibiaxial fracture behavior, we assume with Weibull [6] that  $N$  takes the form

$$N(\sigma_c) = k\sigma_c^m \quad (6)$$

Then

$$P_s(\sigma, \sigma) = \exp[-Vkm \int_0^\infty \left(\frac{\omega}{\pi}\right) \sigma_c^{m-1} d\sigma_c] \quad (7)$$

$$P_s(\sigma, \sigma) = \exp[-Vkm \int_0^\infty \left(\frac{\omega}{\pi}\right) x^{m-1} dx] = \exp[-Vk'x^m] \quad (8)$$

where

$$k' = km \int_0^\infty \frac{\omega}{\pi} x^{m-1} dx \quad (9)$$

$$x \equiv \sigma_c/\sigma \quad (10)$$

The desired relation between uniaxial and equibiaxial fracture statistics can thus be expressed in the form

$$\frac{\ln P_s(\sigma, \sigma)}{\ln P_s(\sigma, \sigma)} = \frac{k}{k'} = \left[ m \int_0^\infty \frac{\omega}{\pi} x^{m-1} dx \right]^{-1} \quad (11)$$

As a simple illustration of the use of (11), consider the case of shear-insensitive cracks. With such cracks, fracture depends only on the normal component of stress  $\sigma_n$  which for simple tension is given by

$$\sigma_n = \sigma \sin^2 \beta \quad (12)$$

Here  $\beta$  is the angle between tensile axis and crack plane. The critical angle  $\beta_c$  occurs when  $\sigma_n = \sigma_c$ , or

$$\beta_c = \sin^{-1} \sqrt{\frac{\sigma_c}{\sigma}} \quad (13)$$

as shown in Fig. 1a.

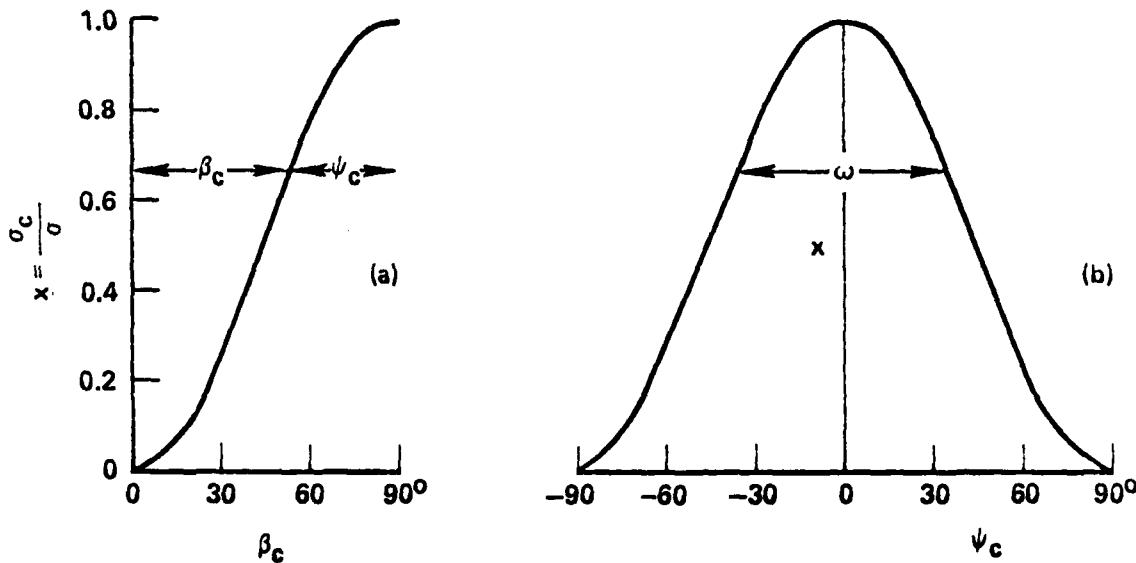


Figure 1. Critical angle and  $\omega$  for shear-insensitive cracks.

The angle  $\omega$  is  $2\psi_c$  where for the case under consideration

$$\psi_c = \cos^{-1} \sqrt{\frac{\sigma_c}{\sigma}} \quad (14)$$

as is evident from inspection of Fig. 1b. Inserting these relations in (11), we obtain

$$\frac{k}{k'} = \left[ \frac{2}{\pi} \int_0^1 \cos^{-1} \sqrt{x} x^{m-1} dx \right]^{-1} = F_1(m) \quad (15)$$

The relation between  $k/k'$  and  $m$  is shown as the top curve in Fig. 2. This curve gives the results of Weibull theory, which tacitly assumes that cracks are shear-insensitive [7].

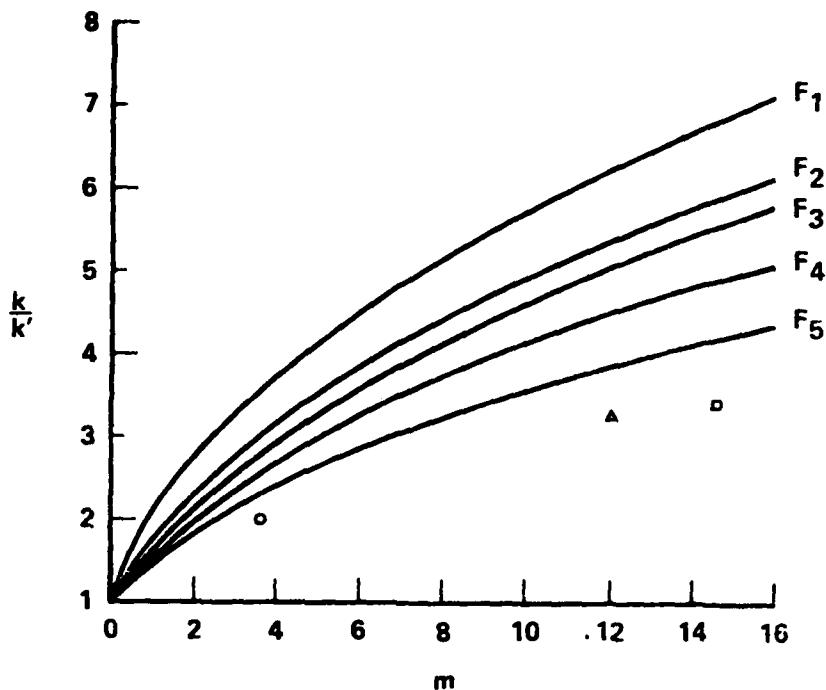


Figure 2. Uniaxial/equibiaxial relation for five fracture criteria.

Other fracture criteria leading to other functional relations between  $\omega$  and  $x$  and therefore other functions  $F_i(m)$  have been discussed in [5] and [8]. For the sake of brevity, we content ourselves here with the identification of crack types and fracture criteria leading to curves  $F_2$  to  $F_5$ , as shown in Table I.

CURVES	CRACK TYPE	FRACTURE CRITERION
$F_2(m)$	Open; Griffith	Tensile stress in crack surface.
$F_3(m)$	Open; Penny-shaped	
$F_4(m)$	Open or closed; Griffith	Energy release rate for co- planar extension.
$F_5(m)$	Open or closed; Penny-shaped	

Table I. Identification of curves in Fig. 2.

The curves for  $F_3$  and  $F_5$  depend on Poisson's ratio  $\nu$ , which was assumed to be 0.25.

In addition to  $F_1$  to  $F_5$ , Fig. 2 shows experimental results for three different materials. The circle represents test results taken on tubes of pyrex glass [9]. The triangle represents test results on ATJS graphite bars and disks spun to failure [10]. In the case of the disks, only the failures originating very near the center were included in the analysis. The square represents results on beams and disks of alumina subjected to bending stress [11]. The test results are summarized in Table I.

Material	$m$	$k/k'$
Pyrex	3.35	2.0
ATJS Graphite	12.00	3.25
Alumina	14.3	3.35

Table II. Experimental  $k/k'$  relation.

It is evident that agreement with the experiment leaves something to be desired. We next turn to predictions based on energy density theory and of fracture criteria strain energy release rate for non-coplanar crack extension.

Sih [12] has shown that for an inclined Griffith crack in simple tension, the critical energy density function is given by

$$S_c = a_{11} k_1^2 + 2a_{12} k_1 k_2 + a_{22} k_2^2 \quad (16)$$

where

$$a_{11} = \frac{1}{16\mu} [3-4v-\cos\theta_0](1+\cos\theta_0) \quad (17a)$$

$$a_{12} = \frac{1}{16\mu} 2\sin\theta_0 [\cos\theta_0 - (1-2v)] \quad (17b)$$

$$a_{22} = \frac{1}{16\mu} [4(1-v)(1-\cos\theta_0) + (1+\cos\theta_0)(3\cos\theta_0 - 1)] \quad (17c)$$

and where  $\mu$  is the shear modulus. For Griffith cracks

$$k_1 = \sigma a^{1/2} \sin^2 \beta \quad (18a)$$

$$k_2 = \sigma a^{1/2} \sin \beta \cos \beta \quad (18b)$$

while for penny-shaped cracks

$$k_1 = \frac{2}{\pi} \sigma a^{1/2} \sin^2 \beta \quad (19a)$$

$$k_2 = \frac{4}{\pi} \sigma a^{1/2} \frac{\sin \beta \cos \beta}{2-v} \quad (19b)$$

$\theta_0$  is chosen by minimizing  $S$  with respect to  $\theta$ .

A plot of the resulting  $x(\beta)$  is shown in Fig. 3. Also shown is a plot of the corresponding relation for penny-shaped cracks.

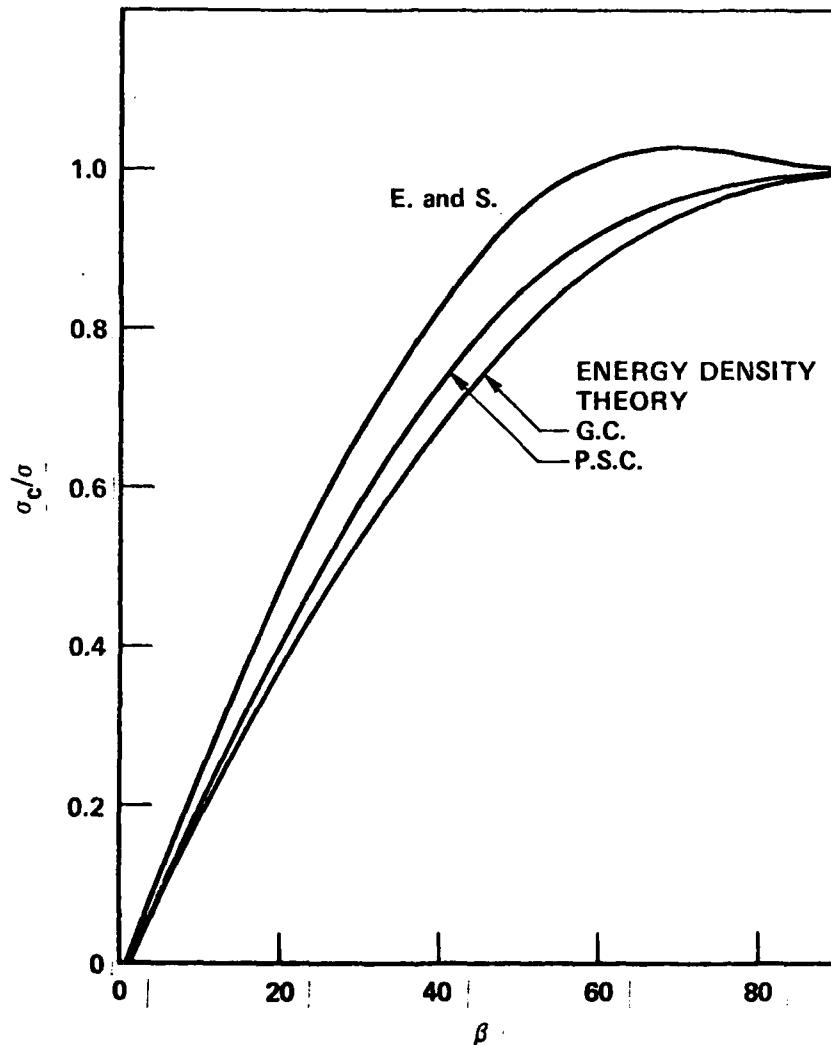


Figure 3. Critical angles for selected fracture criteria.

A major motivation underlying Sih's early efforts in developing energy density theory was the fact that up to that time, mathematical difficulties had precluded accurate calculation of strain energy release rates for non-coplanar crack extension. Recently two solutions have been offered for the case of closed Griffith cracks, one by Palaniswamy and Knauss [14], and one by Wu [15]. The two solutions are in excellent agreement with each other, and lead to virtually the same strength for an angled crack as that obtained from the theory proposed by Erdogan and Sih [16] using Griffith's stress criterion. This strength can be readily calculated from the following relation [17]

$$\frac{\sigma_c}{\sigma} = \cos \frac{\theta_o}{2} \sin \beta \left( \sin \beta \cos^2 \frac{\theta_o}{2} - \frac{3}{2} \cos \beta \sin \theta_o \right) \quad (19)$$

where  $\theta_0$  is found from the equation

$$\tan\beta \sin\theta_0 + 3\cos\theta_0 = 1. \quad (20)$$

The resulting function  $x(\beta) = \sigma_c/\sigma(\beta)$  is shown as the curve in Fig. 3, labeled "E. and S." Note that this curve reaches a maximum value of about 1.03 at  $\beta_c \sim 70^\circ$ , whereas all the others considered here reach a maximum value of  $x = 1$  at  $\beta = 90^\circ$ . This peculiarity has important consequences for the corresponding value of  $k/k'$ .

When the relations of Fig. 3 are used to evaluate  $\omega$  and the results substituted in (11), the curves for  $k/k'$  shown in Fig. 4 are obtained. The energy density criterion leads to fairly good agreement with the experiment in the case of Griffith cracks, and quite good agreement for penny-shaped cracks. The agreement of statistical theory based on the use of available theories for non-coplanar energy release rate is so poor as to be (in the opinion of the present author) unacceptable because it leads to an incorrect variation of  $k/k'$  with  $m$ .

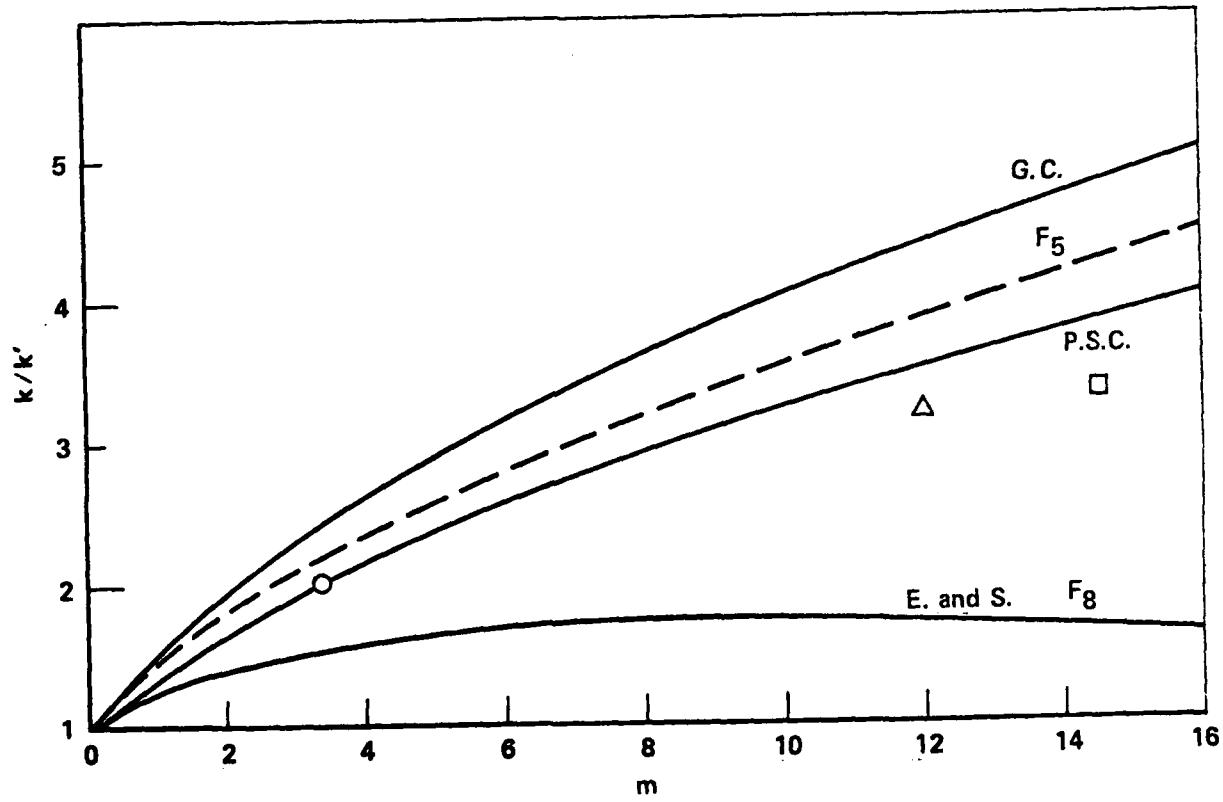


Figure 4. Uniaxial/equibiaxial relation for selected fracture criteria.

## DISCUSSION

The preceding analysis considers open or closed, Griffith or penny-shaped cracks, fracturing according to any one of nine different fracture criteria ( $F_g$  does double duty, representing both the  $\sigma_0$  and the non-coplanar energy release rate criteria). Best agreement with the experiment was obtained using energy density theory as applied to closed penny-shaped cracks. The poor agreement of the relation  $F_g$  with the experiment is quite surprising since it appears to offer in principle an accurate mathematical solution of a physically valid principle (energy balance for non-coplanar crack extension). The low value for the ratio  $k/k'$  can be traced to the influences on the integral in (9) of the portion of the  $x(\beta)$  curve that is greater than unity. This is not only physically implausible, but leads to the incredible prediction that as  $m \rightarrow \infty$ ,  $k/k' \rightarrow 0$ .

It was to be expected that of the two energy density solutions, the one based on use of penny-shaped cracks is in best agreement with the experiment since such cracks are more closely related to naturally occurring cracks than Griffith cracks. To validate the conclusion that it agrees with uniaxial/biaxial strength data better than any other well-known theory, however, it is necessary to justify the use of two-dimensional statistical theory (eq. (11)) rather than the three-dimensional theory that might appear more appropriate [5]. This justification lies in the surprising fact that the two theories lead to the same  $k/k'$  ratio. This was proved in [16] for the relations  $F_1$  and  $F_4$  (see Fig. 2), and has subsequently been shown by the present author to be a generally valid result.

It should be pointed out that the present paper is not the first to discuss statistical fracture theory using energy density theory as the fracture criterion. In a couple of papers published in 1977 [19, 20] Jayatilaka and Trustum considered uniaxial and biaxial stress states, respectively. However, their results are flawed by the use of a crack size distribution, and therefore a functional relation for  $N(\sigma_c)$ , that was determined experimentally for surface cracks in glass. There is no reason to expect that the same law should hold for interior cracks in polycrystals. In fact, it is very likely it would not hold even for another sample of the same glass, since surface flaws in glass are generally considered to be due to handling damage. In addition, no comparison of their results were made with experimental data, or even with predictions of statistical fracture behavior based upon other fracture criteria.

Finally, there is the question of the reliability of the data. The fact that the three data points (a) each represent many tests, (b) involve different types of experiments, (c) were carried out by different experimenters on different materials, and (d) collectively establish a trend closely related to what would be expected on theoretical grounds, tends to establish confidence in their validity. It would nevertheless be highly desirable to obtain additional experimental data to give greater assurance to the conclusions reached in this investigation.

#### CONCLUSIONS

1. The relation between the fracture statistical behavior of specimens in simple tension and that in equibiaxial tension depends on both the assumed crack shape and the fracture criterion employed.
2. The present study investigates the uniaxial/equibiaxial relation for Griffith and penny-shaped cracks for a number of fracture criteria. Best (in fact, very good) agreement with the experiment was obtained assuming energy density (or absorbed specific energy) theory and penny-shaped cracks.
3. Contrary to what might be expected, the maximum energy release rate criterion for non-coplanar extension of Griffith cracks leads to a uniaxial/equibiaxial fracture relation that is in poor agreement with experiment.

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